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A toy model for population dynamics and ecological resilience using Japanese paper balloons

Tomonori Kawano[1,2,3,4,5]

[1] International Photosynthesis Industrialization Research Center, Faculty and Graduate School of Environmental Engineering, The University of Kitakyushu, Kitakyushu, Japan;

[2] University of Florence LINV Kitakyushu Research Center (LINV@Kitakyushu), Kitakyushu, Japan;

[3] Kitakyushu Museum of Natural History and Human History, Kitakyushu, Japan;

[4] RIKEN Center for Advanced Photonics, Wako, Saitama, Japan;

[5] Université Paris Diderot, Sorbonne Paris Cité, Institut des Energies de Demain, Paris, France.

(kawanotom@kitakyu-u.ac.jp)

Abstract

In theoretical physics, ecology and/or social sciences, toy models are oftenly invented for simplifying the complex systems without sacrificing the key features of the original phenomena of interest. Population dynamics are central to ecological studies handling a wide variety of wild species. Resilience, an ecological term, is generally defined as the capacity of an ecosystem to recover from temporal perturbations. In the present article, the scientific aspects of the mechano-stimulated self-inflating nature of Japanese paper toy called *Kamifusen* literally meaning the paper balloon by viewing it as a novel toy model for microscopic and macroscopic resilience and population dynamics are described. In the toy model using a toy, by analogy to the kinetics of popuplation growth of animal and microbes, modified logistic equations were applied to study the mechanically simulated increase in the volume of air inside the paper balloon (V), by viewing the system as the function of the number of mechanical stimuli (n_F) while considering the maximal air volume, intrinsic rate of air increase upon mechano-stimulation, and inflation-enhanced furhter inflation as the value equivalent to carrying capacity (K), growth rate (r), and Allee effect, respectively.

Keywords: Allee effect, Ecology, *Kamifusen*, logistic equation, population dynamics, resilience, toy model

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Introduction

In theoretical physics (Marzuoli, 2008) and ecology or social sciences (Galafassi et al., 2017), toy models are oftenly invented for the purpose to make the modelling of complex systems simpler and for the ease of handling while preserving a few key features of the original phenomena of interest. In most cases in theoretical physics, mathematics plays pivotal roles in development and applications of toy models (Jaffe and Quinn, 1993).

In ecology, the word 'resilience' is generally defined as the capacity of an ecosystem to recover from temporal perturbations or as the ability of the systems of interest to recover from the anthropogenic climate changes occationally, resilience could be altered or lost depending on the environmental status. Accordingly, approach of catastrophic thresholds in natural systems such as climate, ecosystems, and populations of organisms may result in a phenomenon called critical slowing down under which only an increasingly slow recovery from small perturbations is allowed, thus, the resilience is likely lost (Dai et al., 2012).

The author has been searching for the toy models explaining the nature of resilience which would be of central to the ecological and environmental interdisciplinary studies. The candidate models being handled in our group are living aquatic microorganisms such as green paramecia and a non-living ball/balloon-shaped traditional Japanese toy made of paper called *Kamifusen*. Here, the case of *Kamifusen* is described.

Kamifusen as a model for system's resilience

A *Kamifusen* is made up with the strips of transparent thin paper material called glassine put together to form a spherical shape. Glassine is a very thin and smooth type of paper which is air and water resistant (Featonby, 2017). As described elsewhere, a balloon of *Kamifusen* behaves in a counter intuitive way, staying inflated when left alone with its air hole open (Featonby, 2018). This strange behavior of the semi-open system of *Kamifusen* is solely due to the fact that the material (glassine) used is 'plastic' rather than elastic, thus likely maintaining the shape (Featonby, 2017). Again, much more counter intuitively, *Kamifusen* resiliently maintains the volume of inner air while being exposed to the repeated batting (application of mechanical pressure), suggesting that the force pushing the air out of the system is rapidly followed by the recovering phases by gradually letting the external air re-flow into the balloon (Fig. 1). Moreover, the balloon of *Kamifusen* further inflates automatically when struck, especially when the balloon was semi-deflated (semi-inflated) prior to the mechanical challenges. By this way, the phenomenon mimicking the resilience is performed and the amount of air entering the system during the recovering phase has tendency to exceed the amount of air expelled by the preceding mechanical force, thus effectively allowing the swelling of the balloon towards the size limit.

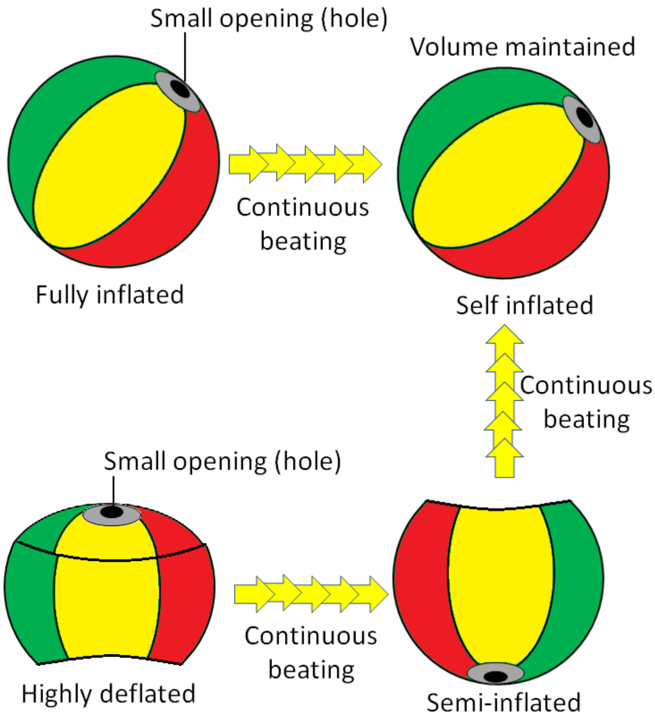

Fig. 1. Self-inflating nature of Japanese paper balloon. Mathematics behind the self-inflating nature of paper balloon must be clarified.

In 2019, two video programs focusing on the scientific aspect of self-inflating nature of *Kamifusen* or Japanese paper balloon (Kid Fun Science, Feb., 18th, 2019; The Action Lab, May 15th, 2019) have been released on YouTube. The video produced by Kid Fun Science simply introduced the self-inflating nature of *Kamifusen*, and the video produced by The Action Lab introduced the likely mathematical model based on the pressure versus time graph and suggested that the spike of elevated pressure inside the balloon is followed by longer phase of lowered pressure (Fig. 2a). In fact, the video by The Action Lab is fully based on the discussion originally brought by Fukumori (2017), focusing on the balloon's oscillation between higher and lower pressure, compared with the external ambient pressure. Fukumori emphasized the fact that the time with lower internal pressure is greater than that with higher internal pressure, so that the net flow of air is inwards, thus leading to swelling of the balloon.

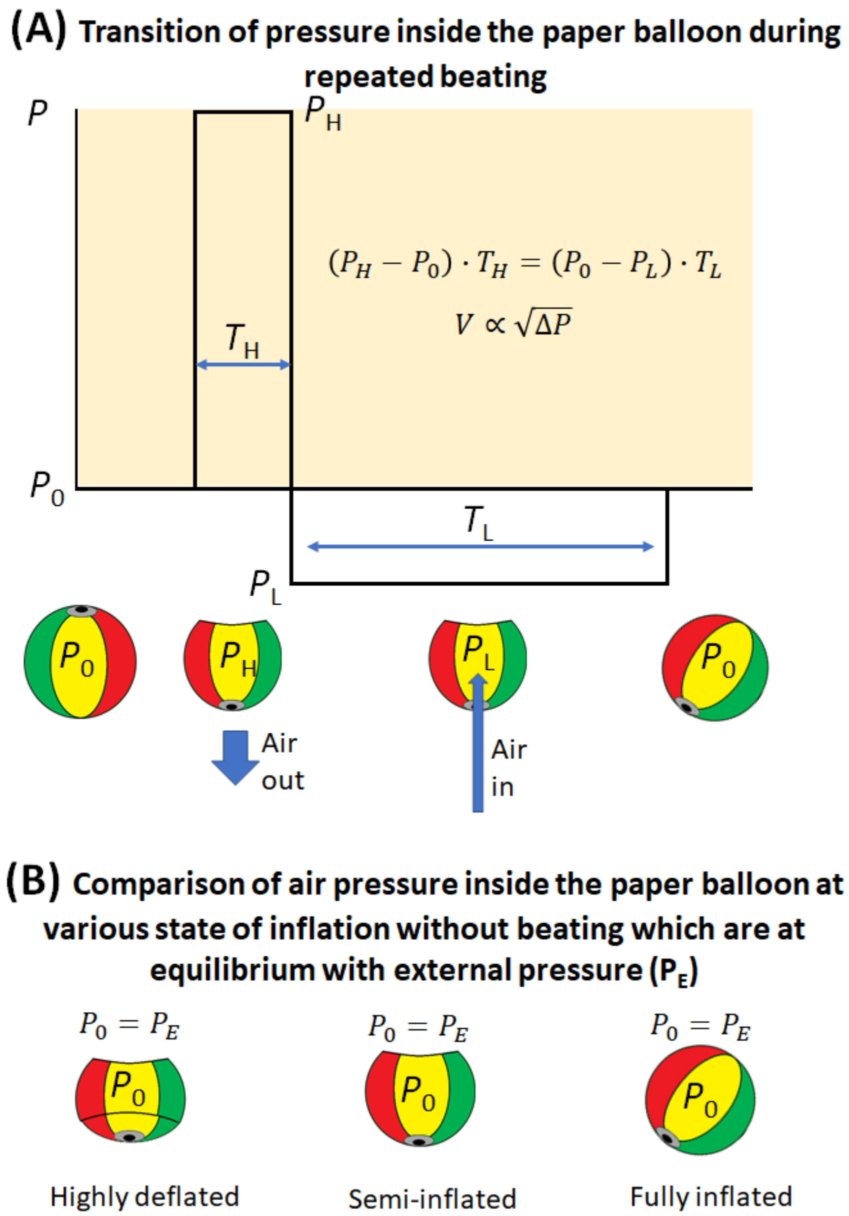

Fig. 2. Changes in the air pressure inside the balloon with and without mechanical forces. (A) Mathematical model proposed in the video by The Action Lab (2019). (B) Comparison of the air pressure inside the balloon at different extent of inflation.

However, due to the plastic nature of the balloon, equilibrium air pressure at static phase (P_0) predicts neither the fate of balloon's volume after next mechanical squashing nor how much air flows in during the self-inflation processes. In fact, P_0 must be at equilibrium with the external air pressure (P_E) nevertheless the extent of inflation (from the fully inflated stage to the fully deflated stage), since the paper balloon is equipped with an opening air-hole, thus the balloon is not a closed or isolated system (Fig. 2b).

Microscopic resilience and possible oscillation of inner air volume

It is highly tempting to focus on the temporal changes in the volume of the air inside the balloon rather than the changes in the pressure (Fig. 3). Under the condition that the serial mechanical forces are applied (continuous batting), the intrinsic forces maintaining the volume of the balloon may include (i) the resistance for the air to flow out through the small opening hole (thus, the resistance is a function of the hole size), (ii) material's plasticity (with minimal elasticity) contributing to the tendency to maintain or form the spherical structure upon increase in inner impact pressure, (iii) partial contribution of rotation to expand the sphere, and so on. This volume-centric model explains the processes for both the maintenance of fully inflated balloon and the swelling of semi-deflated balloon under serial mechanical challenges.

The maximal size of air volume could be simply determined by the size of the fully inflated sphere made of non-stretchable material (sheets of thin papers). Therefore, once the volume of the swelling balloon reaches this size limit, the balloon may stop further swelling but resiliently maintain its size within the range of fluctuation attributed to the depth of spiky drops in volume due to the dynamic impact pressure-dependent acute expelling of air out of balloon and gradual refilling of air. In this study, the above feature is referred to as the microscopic resilience.

Macroscopic resilience and the requirement for the minimal mass of air in the balloon

In our model, we assume that the system made up of paper balloon behaves like a 'natural' computing module performing a series of repeated operation until attaining the equilibrium status, and each operation propelled by a single mechanical input (strike) allows only partial calculation (progress), therefore, for attaining the equilibrium (or to obtain the final outputs), a series of operations propelled by a series of mechanical challenges are required. In this model, each application of the serial mechanical force (F) acts as the driving force for step-wisely achieving a single (thus, partial) operation, by analogy to the 'clock' for processing the operations by a central processing unit (CPU) equipped in computers.

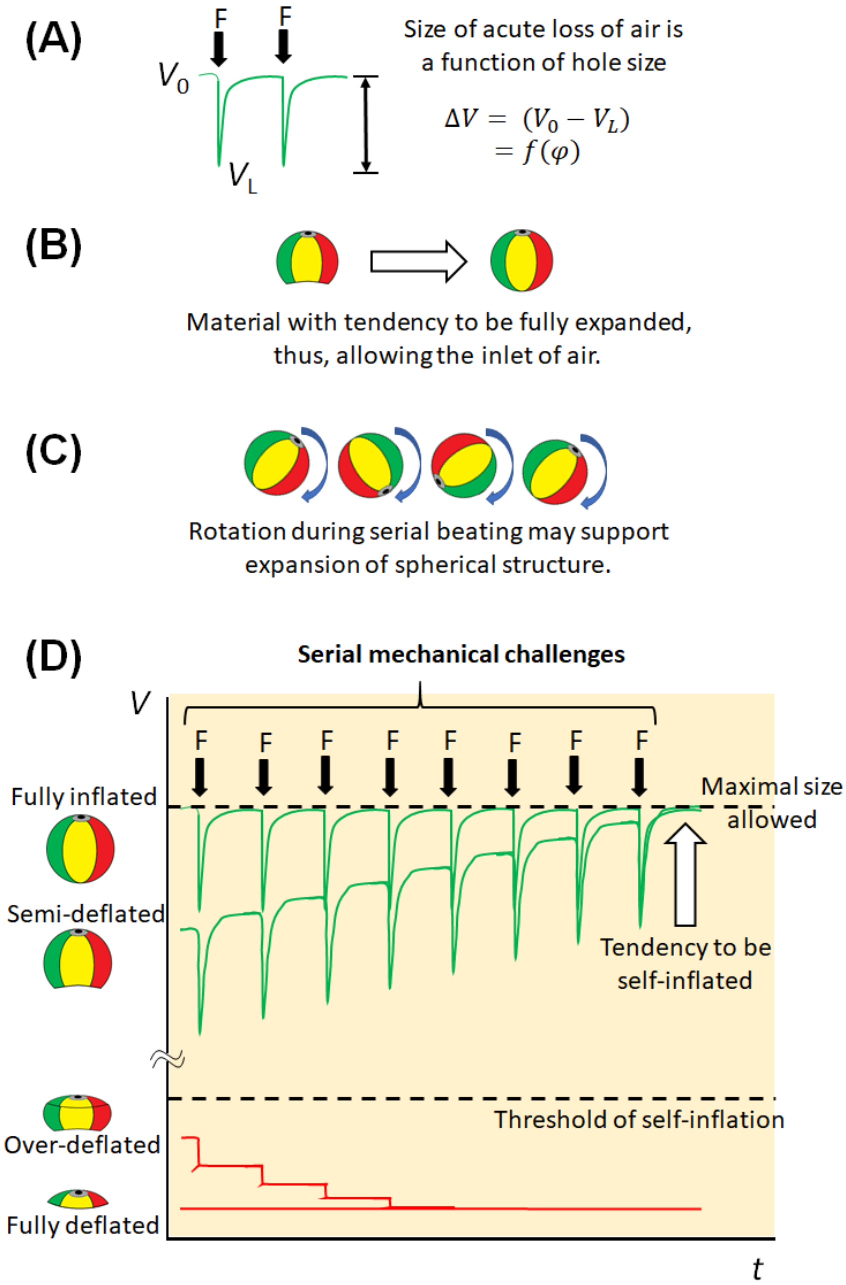

Fig. 3. Changes in the air volume inside the balloon driven by serial application of mechanical force. (A) Resistance to acute deflation could be partially explained by the size of opening limiting the outward flow of air. (B) Material used in paper balloon has tendency to be fully expanded upon mechanical force is added, thus, contributing the inlet air movement. (C) Rotation accompanying the transient centrifugal force due to serial beating may contribute to the expansion of the spherical structure. (D) Schematic diagrams for likely changes in air volume in the balloon under serial mechanical challenges.

By handling the balloons, we noticed that the fully deflated flat sheet of balloon paper no longer performs the self-inflating characteristics, therefore, strongly suggesting that there must be a threshold volume of inner air determining the fate of the balloon to be self-inflated or to be shrunk upon repeatedly struck. It is tempting to assume that the air remaining inside the balloon must have specific role in the self-inflation mechanism. The likely role for the pre-existing air enclosed inside the balloon to be subjected to the self-inflation mechanism, could be to perceive the force (or mechanical impact) and to convert it into the pressure inside the balloon, thus, contributing to the expansion (stretching) of the sheets of paper at the side opposing to the geometric position of external impact. Therefore, the air captured inside the balloon as the mass, behaves as the media for converting the external dynamic impact pressure into the inner impact pressure. Just like the relationship between the investment and the yield, the mass of air captured inside the balloon must be required for the net inlet of air upon strikes.

When the paper balloon is semi-deflated (thus, containing a plenty of air as the media for perceiving the mechanical impact), the serial beatings act for inflating the balloon towards full expansion of the sphere. This feature of the paper balloon must be viewed as the macroscopic resilience recovering from the major loss of air, through accepting the continuous mechanical stimulations. The above nature of the air captured inside the balloon for further allowing the inlet of air upon receiving the dynamic impact pressure by batting largely resembles the population dynamics in ecology or demography as discussed in the section below.

Logistic model applied to the growth of the paper balloon

For expressing the changes in population of various organisms including human being, the logistic model should be employed (Takaichi and Kawano, 2016). Most living organisms including animals and plants show Malthusian-type of drastic growth of population at low populational density, which is reflected by the increase in the number of individuals (N) with intrinsic rate of natural increase (r) over time (t), thus, expressed as below.

$$\frac{dN}{dt} = rN \qquad \text{[Eq. 1]}$$

This type of growth continues until attaining the range affected by the environmental capacity size described as carrying capacity (K), thus, overall growth in population can be expressed by the following logistic equation proposed by Lotka in 1925:

$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right) \qquad \text{[Eq. 2]}$$

When there is a threshold of population size determining the lowest limit of population capable of further population growth, the logistic model should be modified as below:

$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right)\left(\frac{N-A}{A}\right) \qquad \text{[Eq. 3]}$$

where *A* is the critical point known as Allee threshold (Kramer et al., 2009; Alee and Bowen, 1932).

Here, the logistic model originally designed for ecological studies must be modified for handling the case of paper balloon swelling under serial dynamic stimuli, by replacing the population (*N*) and time (*t*) in the logistic growth model with the volume of air inside the balloon (*V*) and the number of mechanical forces applied (n_F), respectively; thus, forming a novel equation as below.

$$\frac{dV}{dn_F} = rV\left(\frac{K-V}{K}\right)\left(\frac{V-A}{A}\right) \qquad \text{[Eq. 4]}$$

Based on the novel logistic model which is the function of the number of dynamic inputs (n_F), the likely curves for the self-inflation of the Japanese paper balloons with different starting points (differed in the extent of initial inflation/deflation) are compared in Fig. 4(A). This model clearly demonstrates the similarity between the population dynamics (as the function of time) and the paper balloon model sharing the innate macroscopic resilience and the growth kinetics determined by the growth rate (*r*), carrying capacity (*K*) and Allee threshold (*A*).

There are several mathematical model for describing the nature of Allee effects (Boukal and Berec, 2002; Ferreira et al., 2013). The above model expressed with Eq. 4 is often referred to as the strong Allee effect model. As shown below, there could be at least two distinct Allee effect models, namely, the flexible Allee effect model [Eq. 5] and the weak Allee effect model [Eq. 6], predicting the presence of a small or no Allee threshold, respectively.

$$\frac{dV}{dn_F} = rV\left(\frac{K-V}{K}\right)\left(\frac{V-A}{K}\right) \qquad \text{[Eq. 5]}$$

$$\frac{dV}{dn_F} = \frac{r}{K}V^2\left(\frac{K-V}{K}\right) \qquad \text{[Eq. 6]}$$

Note that in the last term in Eq. 5 corresponding to the Allee effect, *K* is used as the denominator (instead of *A*). This replacement of *K* with A allows the flexibility of the equation capable of expressing both strong and week Allee effect depending on the size of Allee threshold (*A*). As *A* value approaches zero, the Allee effect becomes week. The Eq. 6 shown below is a special case of flexible Allee effect model in which A is set as zero.

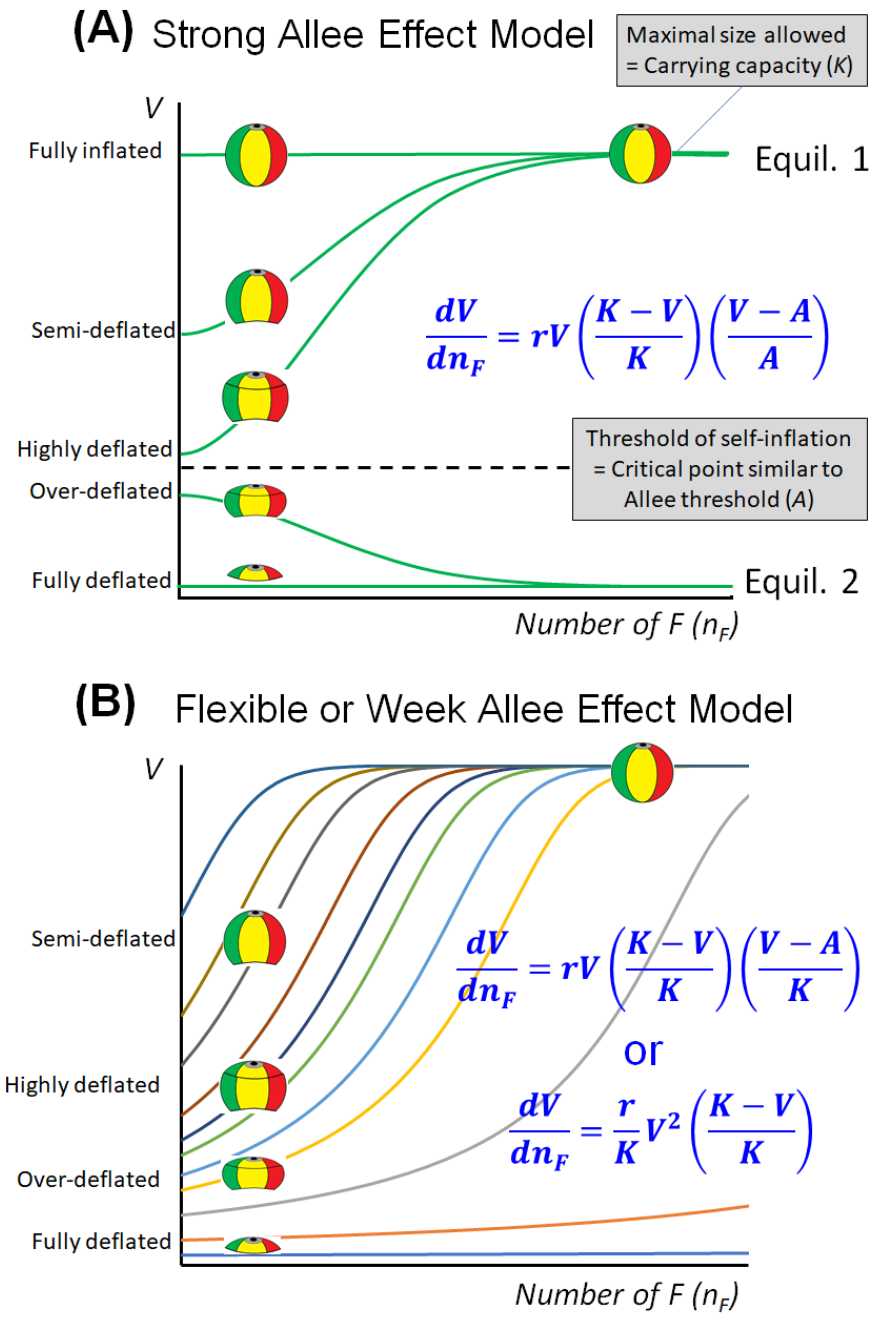

Fig. 4. Possible handling of the paper balloon's growth kinetics using modified Logistic models. (A) A working hypothesis predicting the presence of Allee threshold determining the fate of balloon under repeated batting. (B) Combined weak Allee models explaining the inflation-enhanced further inflation mechanism with low or none Allee threshold.

Perspectives

Although some of our preliminary runs happily played with students in a lecture class likely support the behavior of a small type of Japanese paper available from a stationary shop in Kitakyushu city to be obeying the logistic model with the week Allee effect (Fig. 4B), thus explaining the nature of swelling acceleration as air volume inside the balloon attain semi-deflated level, the author feels like to avoid generalization.

The reliability of the logistic models for the explaining the swelling growth patterns of Japanese paper balloon must be experimentally testified by readers and the factors such as the values for r, A (if any) and K must be determined for each specific case employing your paper balloons.

Furthermore, mathematical analogy between the behavior of Japanese paper balloon and the phenomena in the natural ecosystems (such as resilience and population dynamics) may be beneficial to elucidate the clues to the ecologically problems of interest.

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ポピュレーション・ダイナミクスと生態学的レジリアンスを再現するトイ・モデルとしての日本の紙風船

要約

物理学や生態学の諸分野では、現象の本質を損なわない範囲でシステムの複雑さを回避し、諸問題の理解を容易にするためのトイ・モデルが利用されている。生態学では、様々な生物種を対象とした個体群動態論（ポピュレーションダイナミクス）が盛んに研究されている。また、生態学におけるレジリエンス（しなやかさ）とは、一過的な攪乱に対して生態系が示す回復における許容量を表す概念である。本研究においては、日本の伝統的な玩具である紙風船が示す、繰り返される打撃に応用し内包する空気の容量を一定に保つ性質に着目し、個体群動態論や生態学におけるレジリエンスの概念を再構築し、トイ・モデルとしての利用することを提案する。このトイ・モデルでは、ロジスティック方程式を用いて、取り込まれた空気の体積（V）を個体群密度と同様の挙動をするパラメーターと見立て、体積の増加率を成長率（r）に、最大膨張時の体積を環境許容量（K）に相当する値として扱い、このロジスティックモデルに、Allee 効果が存在することを示した。この過程で、微視的なレジリエンスと巨視的なレジリエンスが数学モデルの中で表現できることも示した。